

6.1 Circles and Circumference

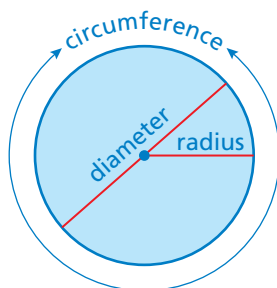
Essential Question How can you find the circumference of a circle?

Archimedes was a Greek mathematician, physicist, engineer, and astronomer.

Archimedes discovered that in any circle the ratio of circumference to diameter is always the same. Archimedes called this ratio pi, or π (a letter from the Greek alphabet).

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

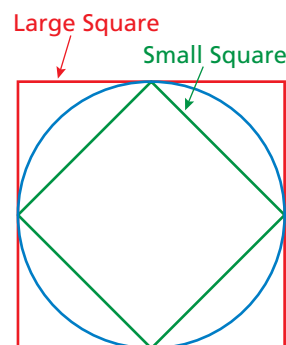
In Activities 1 and 2, you will use the same strategy Archimedes used to approximate π .



1 ACTIVITY: Approximating Pi

Work with a partner. Copy the table. Record your results in the table.

- Measure the perimeter of the large square in millimeters.
- Measure the diameter of the circle in millimeters.
- Measure the perimeter of the small square in millimeters.
- Calculate the ratios of the two perimeters to the diameter.
- The average of these two ratios is an approximation of π .



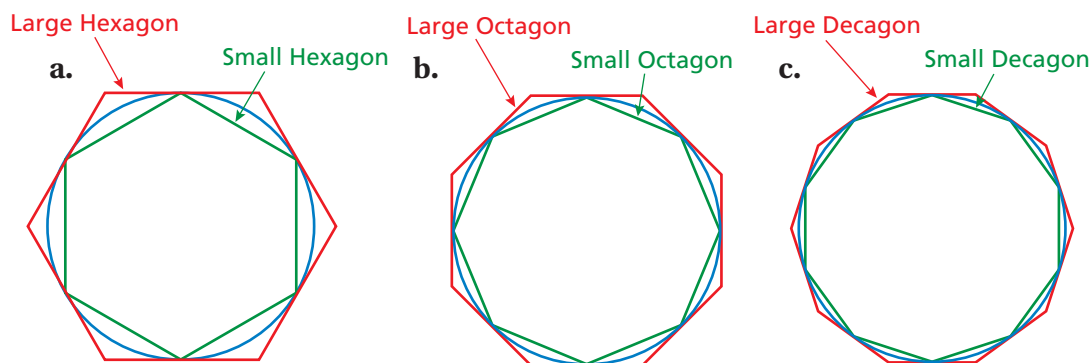
Sides of Polygon	Large Perimeter	Diameter of Circle	Small Perimeter	$\frac{\text{Large Perimeter}}{\text{Diameter}}$	$\frac{\text{Small Perimeter}}{\text{Diameter}}$	Average of Ratios
4						
6						
8						
10						

A page from "Sir Cumference and the First Round Table" by Cindy Neuschwander.



2 ACTIVITY: Approximating Pi

Continue your approximation of pi. Complete the table using a hexagon (6 sides), an octagon (8 sides), and a decagon (10 sides).



- d. From the table, what can you conclude about the value of π ? Explain your reasoning.
- e. Archimedes calculated the value of π using polygons having 96 sides. Do you think his calculations were more or less accurate than yours?

What Is Your Answer?

3. **IN YOUR OWN WORDS** Now that you know an approximation for pi, explain how you can use it to find the circumference of a circle. Write a formula for the circumference C of a circle whose diameter is d . Draw a circle and use your formula to find the circumference.

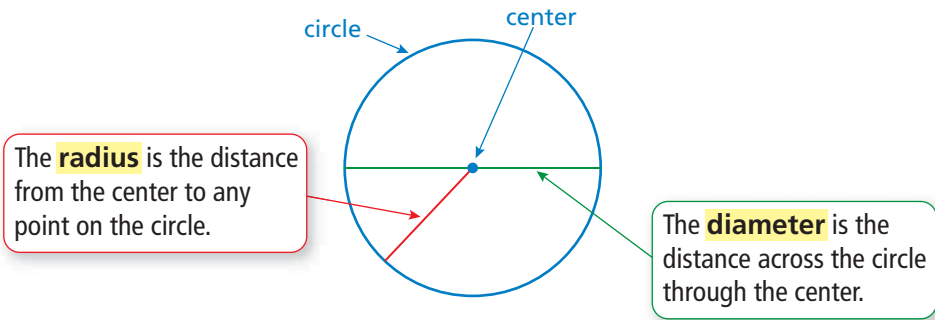


Use what you learned about circles and circumference to complete Exercises 10–12 on page 243.

A **circle** is the set of all points in a plane that are the same distance from a point called the **center**.

Key Vocabulary

circle, p. 240
center, p. 240
radius, p. 240
diameter, p. 240
circumference, p. 241
pi, p. 241
semicircle, p. 242



Key Idea

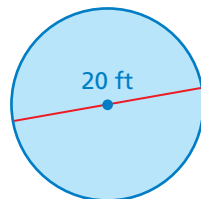
Radius and Diameter

Words The diameter d of a circle is twice the radius r . The radius r of a circle is one-half the diameter d .

Algebra **Diameter:** $d = 2r$ **Radius:** $r = \frac{d}{2}$

EXAMPLE 1 Finding a Radius and a Diameter

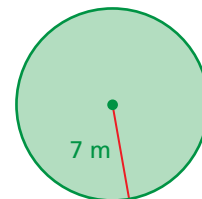
a. The diameter of a circle is 20 feet. Find the radius.



$$\begin{aligned} r &= \frac{d}{2} && \text{Radius of a circle} \\ &= \frac{20}{2} && \text{Substitute 20 for } d. \\ &= 10 && \text{Divide.} \end{aligned}$$

∴ The radius is 10 feet.

b. The radius of a circle is 7 meters. Find the diameter.



$$\begin{aligned} d &= 2r && \text{Diameter of a circle} \\ &= 2(7) && \text{Substitute 7 for } r. \\ &= 14 && \text{Multiply.} \end{aligned}$$

∴ The diameter is 14 meters.

On Your Own

Now You're Ready
Exercises 4–9

- The diameter of a circle is 16 centimeters. Find the radius.
- The radius of a circle is 9 yards. Find the diameter.

The distance around a circle is called the **circumference**. The ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for *every* circle and is represented by the Greek letter π , called **pi**. The value of π can be approximated as 3.14 or $\frac{22}{7}$.

Study Tip

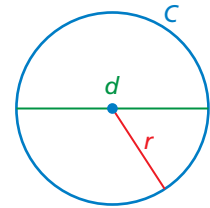
When the radius or diameter is a multiple of 7, it is easier to use $\frac{22}{7}$ as the estimate of π .

Key Idea

Circumference of a Circle

Words The circumference C of a circle is equal to π times the diameter d or π times twice the radius r .

Algebra $C = \pi d$ or $C = 2\pi r$



EXAMPLE 2 Finding Circumferences of Circles



a. Find the circumference of the flying disc. Use 3.14 for π .

$$\begin{aligned} C &= 2\pi r && \text{Write formula for circumference.} \\ &\approx 2 \cdot 3.14 \cdot 5 && \text{Substitute 3.14 for } \pi \text{ and 5 for } r. \\ &= 31.4 && \text{Multiply.} \end{aligned}$$

∴ The circumference is about 31.4 inches.



b. Find the circumference of the watch face. Use $\frac{22}{7}$ for π .

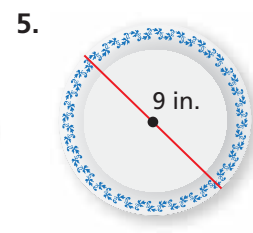
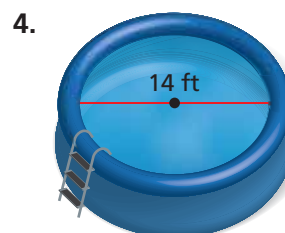
$$\begin{aligned} C &= \pi d && \text{Write formula for circumference.} \\ &\approx \frac{22}{7} \cdot 28 && \text{Substitute } \frac{22}{7} \text{ for } \pi \text{ and 28 for } d. \\ &= 88 && \text{Multiply.} \end{aligned}$$

∴ The circumference is about 88 millimeters.

On Your Own

Now You're Ready
Exercises 10–13

Find the circumference of the object. Use 3.14 or $\frac{22}{7}$ for π .



EXAMPLE 3 Standardized Test Practice

The diameter of the new roll of caution tape decreases 3.25 inches after a construction worker uses some of the tape. Which is the best estimate of the circumference of the roll after the decrease?

- (A) 9 inches (B) 16 inches (C) 21 inches (D) 30 inches

After the decrease, the diameter of the roll is $10 - 3.25 = 6.75$ inches.

$$\begin{aligned}
 C &= \pi d && \text{Write formula for circumference.} \\
 &\approx 3.14 \cdot 6.75 && \text{Substitute 3.14 for } \pi \text{ and 6.75 for } d. \\
 &\approx 3 \cdot 7 && \text{Round 3.14 down to 3. Round 6.75 up to 7.} \\
 &= 21 && \text{Multiply.}
 \end{aligned}$$

∴ The correct answer is (C).

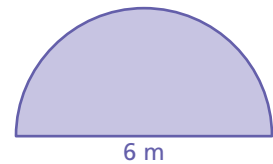
On Your Own

6. **WHAT IF?** In Example 3, the diameter of the roll of tape decreases 5.75 inches. Estimate the circumference after the decrease.

EXAMPLE 4 Finding the Perimeter of a Semicircular Region

A **semicircle** is one-half of a circle. Find the perimeter of the semicircular region.

The straight side is 6 meters long. The distance around the curved part is half the circumference of a circle with a diameter of 6 meters.



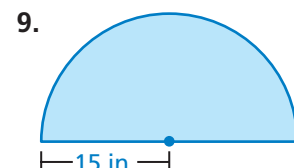
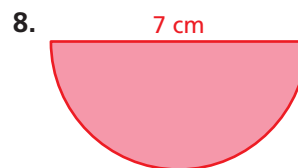
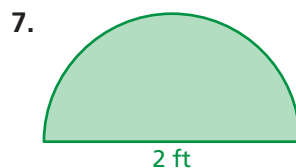
$$\begin{aligned}
 C &= \frac{\pi d}{2} && \text{Divide the circumference by 2.} \\
 &\approx \frac{3.14 \cdot 6}{2} && \text{Substitute 3.14 for } \pi \text{ and 6 for } d. \\
 &= 9.42 && \text{Simplify.}
 \end{aligned}$$

∴ So, the perimeter is about $6 + 9.42 = 15.42$ meters.

On Your Own

Find the perimeter of the semicircular region.

Now You're Ready
Exercises 15 and 16



Vocabulary and Concept Check

- VOCABULARY** What is the relationship between the radius and the diameter of a circle?
- WHICH ONE DOESN'T BELONG?** Which phrase does *not* belong with the other three? Explain your reasoning.

the distance around a circle

π times twice the radius

π times the diameter

the distance from the center to any point on the circle

- OPEN-ENDED** Choose a real-life circular object. Explain why you might need to know its circumference. Then find the circumference.

Practice and Problem Solving

Find the radius of the button.

1

4.



5.



6.

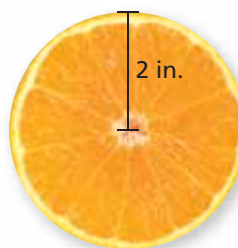


Find the diameter of the object.

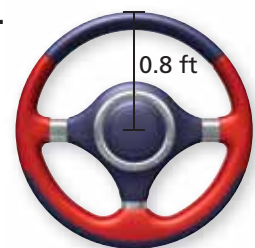
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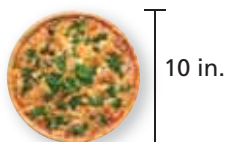
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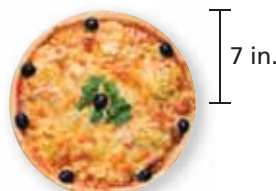
Find the circumference of the pizza. Use 3.14 or $\frac{22}{7}$ for π .

2

10.



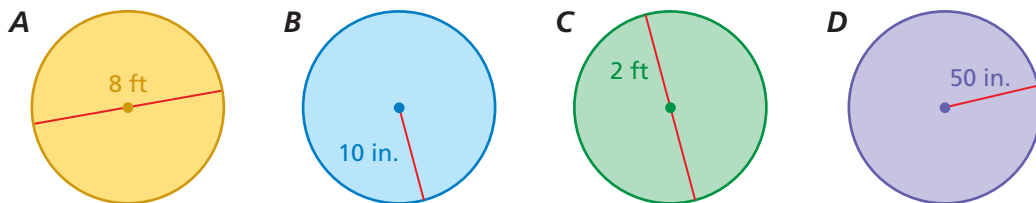
11.



12.



13. **SINKHOLE** A circular sinkhole has a radius of 12 meters. A week later, it has a diameter of 48 meters. How much greater is the circumference of the sinkhole compared to the previous week?
14. **REASONING** Consider the circles *A*, *B*, *C*, and *D*.

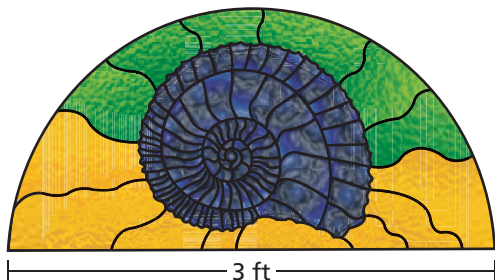


- a. Copy and complete the table.
 b. Which circle has the greatest circumference?
 c. Which circle has the least circumference?

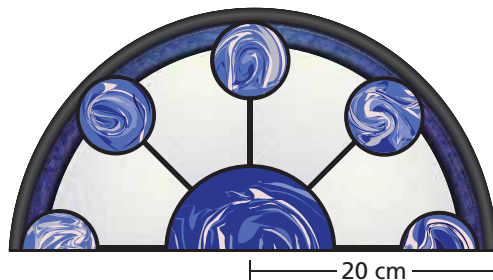
Circle	A	B	C	D
Radius		10 inches		50 inches
Diameter	8 feet		2 feet	

Find the perimeter of the window.

4 15.

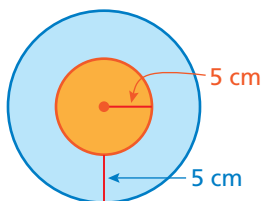


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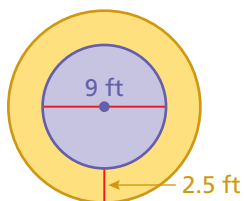


Find the circumferences of both circles.

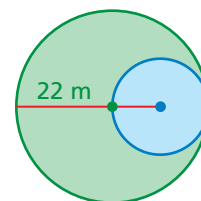
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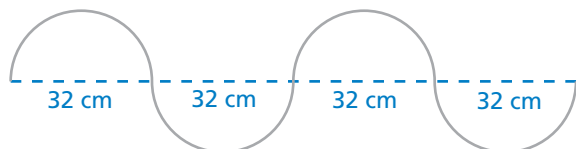
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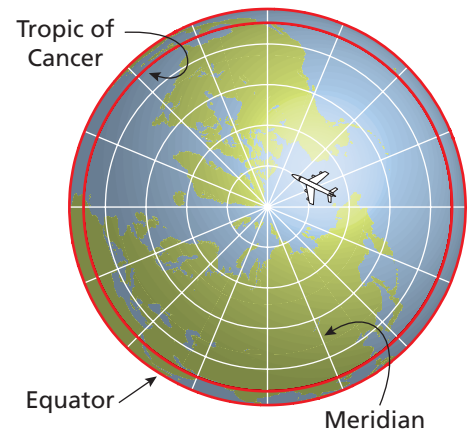


20. **WIRE** A wire is bent to form four semicircles. How long is the wire?

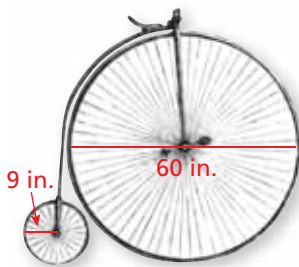


21. **CRITICAL THINKING** Because the ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for every circle, is the ratio $\frac{\text{circumference}}{\text{radius}}$ the same for every circle? Explain.

22. **AROUND THE WORLD** “Lines” of latitude on Earth are actually circles. The Tropic of Cancer is the northernmost line of latitude at which the Sun appears directly overhead at noon. The Tropic of Cancer has a radius of 5854 kilometers. To qualify for an around-the-world speed record, a pilot must cover a distance no less than the circumference of the Tropic of Cancer, cross all meridians, and end on the same airfield where he started.



- What is the minimum distance that a pilot must fly to qualify for an around-the-world speed record?
- RESEARCH** Estimate the time it would take for a pilot to qualify for the speed record.

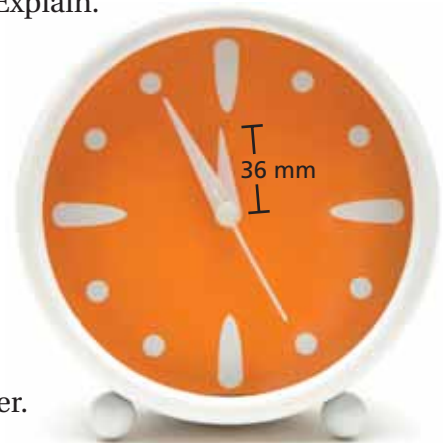


23. **BICYCLE** Bicycles in the late 1800s looked very different than they do today.

- How many rotations does each tire make after traveling 600 feet? Round your answers to the nearest whole number.
- Would you rather ride a bicycle made with two large wheels or two small wheels? Explain.

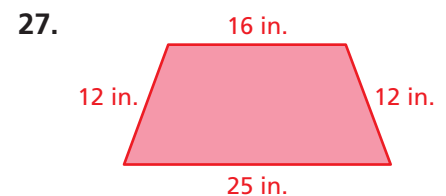
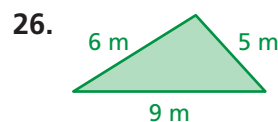
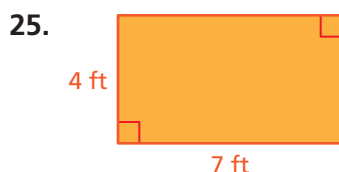
24. **Critical Thinking** The length of the minute hand is 150% of the length of the hour hand.

- What distance will the tip of the minute hand move in 45 minutes? Explain how you found your answer.
- In 1 hour, how much farther does the tip of the minute hand move than the tip of the hour hand? Explain how you found your answer.



Fair Game Review what you learned in previous grades & lessons

Find the perimeter of the figure.



28. **MULTIPLE CHOICE** What is the median of the data set?

12, 25, 16, 9, 5, 22, 27, 20

- (A) 7 (B) 16 (C) 17 (D) 18